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The Leadership Program in Discrete Mathematics

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1. Introduction to the Leadership Program

During the period from 1989 to 1998, the *Leadership Program in Discrete Mathematics* will have involved about 1000 K–12 teachers in an intensive and exciting introduction to discrete mathematics. In this article we will describe the *Leadership Program* (LP) and the lessons that we have learned from it. We will also describe the ways in which the LP serves as a continuing resource to teachers who have not participated in the program, as well as those who have.

A. History of the Leadership Program. The story of the *Leadership Program in Discrete Mathematics* begins with a proposal to the National Science Foundation in 1988 for funding the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS)[1] as a Science and Technology Center (STC). The proposal included a provision that DIMACS would support programs for teachers and students in collaboration with the Rutgers University Center for Mathematics, Science, and Computer Education (CMSCE). Soon after NSF announced in February 1989 that DIMACS would receive the STC award, planning began for a summer program for teachers. This two-week program, entitled *Networks and Algorithms*, took place in the summer of 1989 with 27 high school teachers; it was funded entirely by DIMACS.

1991 *Mathematics Subject Classification.* Primary 00A05, 00A35.

Joseph G. Rosenstein has served as Director of the Leadership Program in Discrete Mathematics since its inception in 1989. Valerie A. DeBellis has also been associated with the LP since its inception, and has served as its Associate Director since 1992.

[1]DIMACS is an NSF-funded Science and Technology Center which was founded in 1989 as a consortium of Rutgers and Princeton Universities, AT&T Bell Laboratories, and Bellcore (Bell Communications Research). With the reorganization of AT&T Bell Laboratories in 1996, it was replaced in the DIMACS consortium by AT&T Labs and Bell Labs (part of Lucent Technologies). DIMACS is also funded by the New Jersey

The 1989 *Networks and Algorithms* institute served as a pilot for a more ambitious program that was first funded by NSF the following year — the *Leadership Program in Discrete Mathematics.* Initially, the LP was funded by NSF for two years as an institute for high school teachers. Then, as the LP — Phase II, it was funded by NSF for three years as an institute for high school and middle school teachers. Finally, as the LP — Phase III, it was funded by NSF for four years as an institute for K–8 teachers. Each program also received financial support from DIMACS and from Rutgers University, and each program was co-sponsored by DIMACS and CMSCE.

B. Size of the Leadership Program. A clear progression over time has been the increase in the size of the program. During Phase I of the LP (of course, it was not referred to as Phase I before Phase II was funded), there were about thirty-five participants during each of 1990 and 1991. During Phase II, which involved parallel institutes for high school and middle school teachers, there were about eighty teachers during each of 1992, 1993, and 1994. During Phase III, there were three institutes for K–8 teachers with 120 participants in 1995, and, in each of 1996, 1997, and 1998, five institutes with 180 participants. Two of the Phase III institutes each year are residential institutes at Rutgers, and the other three are commuter institutes, one at Rutgers and two at other sites; the "off-site" institutes in 1996 were in Rhode Island and Virginia, and in 1997 will be in Rhode Island and Arizona. Including the pilot institute in 1989, the total number of participants is approximately 1000. The scope of Phase III of the LP can be seen by reviewing the schedule for the summer of 1997. There are five two-week institutes for teachers who are new to the program, five one-week institutes for teachers in the 1996 cohort who are returning for their second summer's activities, two one-week institutes for teachers from 1989–1995, and a two-day "crash course" for high school teachers. Altogether, there are over seventeen weeks of institutes during the summer, with an anticipated total attendance of over 400 teachers.

C. The evolving target audience of the Leadership Program. Another clear progression over time has been that the participants have been teachers of progressively younger and younger students. This was not the intention at the outset, but it reflected what we learned from the participants in the program about the suitability and the value of introducing discrete mathematics to students of all grade levels and all ability levels.

The pilot program was targeted to teachers of high achieving seniors, since we thought that it was for those students that discrete mathematics was most appropriate. Many of the participants in the pilot

We soon learned that other students were able to benefit from exposure to discrete mathematics as much, if not more than the high achieving students, and so our assumption became that teachers in the program would be introducing discrete mathematics to average high school students. Then we learned that students who had been unsuccessful with traditional mathematics could be successful with discrete mathematics — partly because of the visual, geometric component of the topics in discrete mathematics that were the focus of the program, and partly because discrete mathematics did not require a strong background in the mathematics with which they had been unsuccessful.

This realization, that discrete mathematics could provide a "new start" for students, led to the October 1992 conference *Discrete Mathematics in the Schools: How Can We Make an Impact?* which in turn led to the publication of this volume. It first appeared in a "concept document" developed by Joseph Rosenstein in January 1991, and was then incorporated into the charge to the conference, a revised version of which appears as the **Introduction**[2] to this volume.

We soon realized that many of the topics which we discussed in the program were equally appropriate for middle school students. As a result, Phase II of the LP included a middle school component. Each summer from 1992 to 1994, the LP involved two parallel institutes, one for forty high school teachers and one for forty middle school teachers. The program for middle school teachers was conducted by faculty from Montclair State College (now Montclair State University). As was the case earlier, we learned from the teachers in the program that discrete mathematics is also appropriate for students at earlier grade levels. Phase III accordingly is addressed to elementary school teachers as well as middle school teachers. We have found that most of the topics in discrete mathematics that are included in the program can indeed be introduced to students at all grade levels, although of course the way in which the topics are introduced may differ considerably between grade levels.

D. Participation in the Leadership Program. Participants in the LP have typically been expected to attend a two- or three-week institute in the first summer, up to four follow-up sessions during the following school year, and a one- or two-week institute the following year. Including the follow-up sessions, each fully-participating teacher has been involved in four to six weeks of discrete mathematics workshops during a period of a year (including two summers), five weeks for teachers in Phase I, six weeks for teachers in Phase II, and four weeks for teachers in Phase III. With the sup-

continue to attend follow-up sessions after their year as official participants in the program, to attend "veterans institutes" in subsequent summers, and to communicate with each other on an active email network.

Participants in the LP are expected to introduce discrete mathematics in their classrooms, incorporate discrete mathematics into their schools' curricula, and introduce their colleagues, both locally and broadly, to topics in discrete mathematics. A substantial percentage of LP participants have fulfilled these expectations and have remained active in LP activities beyond their formal affiliation with the program.

E. Instructional Staff of the Leadership Program. An important feature of the program has been the participation in the instructional staff of college faculty, including both researchers and mathematics educators. This has enabled participants to be in contact with real-live mathematicians and computer scientists, to learn how people working in the field think about their subject, and to experience the mathematical sciences as living disciplines.

Among the faculty members who have conducted workshops extending a week or more have been Ravi Boppana (Rutgers), Margaret Cozzens (Northeastern), Valerie DeBellis (Rutgers), Deborah Franzblau (Rutgers), Robert Garfunkel (Montclair), Robert Hochberg (Rutgers), Glenn Hurlburt (Arizona State), Robert Jamison (Clemson), Kenneth Kaplan (Rutgers), Laura Kelleher (Massachusetts Maritime Academy), Rochelle Leibowitz (Wheaton College — MA), Evan Maletsky (Montclair), Joseph Malkevitch (York College — CUNY), Terence Perciante (Wheaton College — IL), Anthony Piccolino (Montclair), Fred Roberts (Rutgers), Joseph Rosenstein (Rutgers), Donald Smith (Rutgers), Diane Souvaine (Rutgers), Ann Trenk (Wellesley), Tom Trotter (Arizona State), and Kenneth Wolff (Montclair). (Note that affiliations given are those at the time of participation in the LP.)

Many other researchers and educators have visited and participated in the LP, including Steven Brams, Doug Clements, John Conway, Dannie Durand, Nate Dean, Ron Graham, Stuart Haber, Eric Hart, David Johnson, Stephen Maurer, Michael Merritt, Peter Winkler, and Ann Yasuhara. Of special note is Michael Fellows (University of British Columbia), who showed how research problems in computer science can often be brought down to a level which second-graders can understand.

Also of note is the interplay in the formulation and development of the LP between a mathematician (Joseph G. Rosenstein) and a mathematics educator (Valerie A. DeBellis). As Director and Associate Director of the LP,

nie Katz, and Stephanie Micale who have served the LP for many years as Evaluation Coordinator, Program Coordinator, and Secretary, respectively.

2. Broader Goals of the Leadership Program

The goals of the LP are not defined exclusively in terms of the accomplishments of the participants in the area of discrete mathematics, but also in terms of their attitudes and understandings toward mathematics and the teaching and learning of mathematics. By 1991, we had learned that discrete mathematics was not just another interesting area of mathematics which teachers could use in their classrooms, but that it was also an excellent vehicle for changing mathematics education. This was reflected in the "concept document" referred to above: "Discrete mathematics provides an opportunity to focus on *how* mathematics is taught, on giving teachers new ways of looking at mathematics and new ways of making it accessible to their students. *From this perspective, teaching discrete mathematics in the schools is not an end in itself, but a tool for reforming mathematics education.*" [5] The broader goals reflected in this passage are explored in the following paragraphs.

A. Changing participants' attitudes about mathematics. Teachers often view mathematics exclusively as a body of knowledge, as a set of facts, which it is their job to transmit to their students; this should not be surprising since, after all, this has likely been their experience in learning mathematics. It should also not be surprising that, as a result, many students attribute their lack of success in mathematics to their inability to remember all of the required facts, formulas, and techniques.[3]

We would like teachers to view mathematics in terms of reasoning and problem-solving; in order to do that we must expect teachers to reason and solve problems. We would like teachers to recognize the applications of mathematics to the world; in order to do that we must show them how to wear eyeglasses through which they can see the world mathematically.[4] Wrestling with a mathematical situation, what mathematicians would call "doing" mathematics, is not something with which many teachers are familiar; we need to introduce them to the idea of doing mathematics, and foster the idea that they themselves can function as mathematicians, as can their students.[5] And, as educators of teachers, we need to provide teachers with a

[3]When Rosenstein interviewed Rutgers students in his "Mathematics for Liberal Arts" class, many responded independently that they were unsuccessful in mathematics in high school because of their inability to memorize all the formulas and proofs.

supportive learning environment so that they will be comfortable with doing mathematics.

Discrete mathematics is a particularly appropriate environment for enabling teachers to function as mathematicians; as a result, all of these goals have been features of the LP. Participants in the LP are expected to solve problems. These problems go beyond the warm-up exercises that simply test their recall of the workshop topics, although such exercises facilitate a gradual transition from easier to more difficult problems. After each morning workshop, they spend an hour in study groups grappling with the problems, and discuss their solutions in the homework review session the next morning; the daily problems also are the focus of many evening discussions among participants in the residential programs. Discrete mathematics is an area where problems can be concisely stated and easily understood, no matter whether their solutions are simple or difficult, or even if their solutions are unknown. Since many topics in discrete mathematics are connected with real world situations, participants can learn to wear mathematical eyeglasses, seeing applications of graphs, counting, and algorithms all around them. Discrete mathematics lends itself readily to exploration, enabling participants to rediscover principles that mathematicians refer to as theorems. Moreover, since most teachers have had no exposure to topics in discrete mathematics such as graphs (the kind with vertices and edges) and since these topics have few mathematical prerequisities, all participants start on a "level playing field". This makes it possible for early elementary teachers with little mathematical background to work together in a supportive environment with middle school teachers who are more familiar with traditional topics in mathematics; despite their differences in background, it is not uncommon in this type of environment for primary teachers to grasp the essence of a situation before their middle school colleagues.

B. Learning mathematics. The high expectations that we have created for participants are reflected in the schedule of the institute itself. Half of each day is devoted to learning mathematics, and the other half to introducing that mathematics into K–8 classrooms. Each morning, participants are involved in a two-hour content-based workshop on new mathematical topics. This is followed by a one-hour study session in which participants work in small groups on a set of "homework" problems based on the topic of the workshop; before the morning workshop on the next day, they will present solutions to the entire group. The schedule of the follow-up sessions is similar.

Altogether there are twenty-one workshops for the K–8 LP participants;

applications, and the second week focuses on patterns in numbers and geometry. The second summer institute focuses on games and probability. The topics of the workshops at the follow-up sessions are a variety of significant topics in discrete mathematics which are independent of each other and of the topics of the summer workshops. Following are the titles of all the workshops:

First summer

1. Coloring Maps & Resolving Conflicts
2. Drawing Pictures with One Line: Euler Circuits
3. Hamilton Circuits & the Traveling Salesperson Problem
4. Making the Right Connections: Spanning Trees and Algorithms
5. Shortest Routes
6. Introduction to Systematic Counting
7. Combinatorics and Pascal's Triangle
8. Iteration and Recursion
9. Patterns in Geometry
10. Generating Fractals

Follow-up sessions

11. Voting: Consolidating Individual Preferences
12. Codes: Error Detection and Error Correction
13. Fair Division
14. Number Patterns in Nature (including Fibonacci numbers)
15. Directed Graphs and Tournaments
16. Alphabetizing and Sorting

Second summer

17. Paths and Matchings
18. Matchings and Games
19. Games and Strategies
20. Probability
21. Probability and Games

The workshops for high school and middle school teachers in Phase I and Phase II of the LP addressed similar topics, although because high school and middle school teachers typically have more experience in mathematics, these topics could be discussed at greater depth and additional topics could be introduced. During Phase II, when participants attended three weeks in the first summer and two weeks in the second summer, the five weeks focused on the following themes:

1. graphs and their applications
2. algorithms for graphs

The description of discrete mathematics presented in the article in this volume entitled "A Comprehensive View of Discrete Mathematics: Chapter 14 of the New Jersey Mathematics Curriculum Framework" [6] reflects in part the activities used by LP participants in their classrooms. The first draft of that article was drawn from a "content map" which participants in the 1994 veterans program were asked to help develop based on their classroom experiences; the "content map" was designed to indicate classroom activities appropriate for different grade levels and to trace the development of topics in discrete mathematics across the different grade levels.

C. Changing instructional practices. Although each morning at the LP is devoted to mathematical content, the way in which that content is delivered is designed to convey messages about mathematical instruction. We consciously model the behaviors that we would like teachers to carry into their own classrooms, the types of mathematics instruction recommended by the NCTM Standards. Some of these behaviors are described in the following paragraphs.

Using a variety of instructional formats. The morning workshops involve a mixture of whole-group instruction and small-group activity. The pattern that is repeated throughout each workshop involves introduction of new content material, participants' working on a problem, and discussion of the problem and the material. For homework review sessions, the whole group is divided into two smaller groups. Seating in workshop groups in the K–8 program is heterogeneous, with teachers from different grade levels and with different mathematical backgrounds working together; seating in classroom implementation groups in the afternoon is homogeneous, with teachers working with colleagues who deal with children at similar ages. Participants leave the institute with models of introducing new mathematical material which serve as alternatives to the lecture method.

Working in groups on problem-solving. Solving problems in groups provides powerful lessons for all participants, even for those who had been using groups in their own classrooms, because they typically had never themselves learned content material in a group setting. This is facilitated by having participants working at round tables which are conducive to small group interaction. Participants learn about the power of discussion in assisting mathematical learning. They learn about the advantages of working in a group where different participants bring different perspectives and strengths to the problem-solving process; as noted above, each group of teachers in the K–8 program typically includes teachers from all grade levels. They learn about how to achieve the goal of ensuring that everyone in the group has

coaches during the problem-solving activities, not providing answers but raising pertinent questions, suggesting possible directions, and reinforcing participants' confidence in their ability to solve problems themselves. They also conduct the homework review sessions, and the sessions on classroom implementation, including presentations of their own classroom activities with discrete mathematics. The presence of the lead teachers not only facilitates the learning, but also provides strong role models for the future achievement of program participants. Participants are very aware that the lead teachers were introduced to discrete mathematics only a few years ago, and that now they are serving in a leadership capacity. Over 40 teachers have served in this leadership role during the course of the program. Each lead teacher has used discrete mathematics extensively in his or her classroom, has made presentations on her or his classroom experiences to colleagues, including presentations during the summer and follow-up programs, and has served as a coach to participants in the program, both during the institute and subsequently.

Journal writing. A more recent addition to the program is the use of journal writing, with continued feedback from staff, to enhance the mathematical learning of participants. Participants are provided with a ten-page "journal" in which they make daily entries regarding their mathematical learning; this gives participants an opportunity to describe their understanding of the new material and to highlight areas where they are having difficulty with the material. Journals are collected near the end of each day and are reviewed by lead teachers, who respond daily (in writing) to the entries. Journals serve as a way of assessing the program as well as the learning of individual participants. If patterns are found among the journals, the lead teachers respond collectively to the group the following day, and convey the participants' areas of difficulty to the workshop leaders.

Providing opportunities for reflection. After modeling each instructional strategy, participants are asked to reflect on that strategy in organized discussions; this enables them to better understand the strategy and how it can be used. This mode of "modeling then reflecting"[7] on desired behavior is now utilized in a number of contexts, including group learning, problem-solving, journal writing, assessment, and developing an equitable learning environment. Regular opportunities are incorporated into the program to allow participants time to reflect on the institute experience. For example, after working in groups for several days, participants are asked to reflect on what it means to engage in group work and what are important aspects to remember about participating in group learning environments.

D. Challenging the participants. Many teachers who come to professional development activities are looking primarily for activities they can

[7]Use of this mode of "modeling then reflecting" in the LP was stimulated by Eric Hart, who often encouraged us to reflect on this mode as well as model it. As a result we expanded its use, which gave us more to reflect on.

do in their classrooms; they want to be given things that will interest, occupy, and challenge their students. Typically, they are not looking to be challenged themselves. The conflict between these two perspectives emerges by the third day of each institute, when some participants, feeling the challenge keenly, raise the question of "why do we need to learn this, we are only teaching at the x'th grade level", and others, beginning to yield to old negative attitudes about mathematics, decide that they will never overcome the challenges that the LP presents.

The LP is designed to challenge participants to learn mathematics by doing mathematics, that is, by solving mathematical problems whose answers and solution methods they do not know in advance. They need to understand that learning often involves dealing with situations that are challenging, and with concepts that appear inpenetrable; they need to experience frustration when a problem appears insoluble and excitement when it has finally been overcome. Not only do they better understand the mathematical themes and strategies involved in the problem and gain confidence in their mathematical abilities, they also understand the difficulties their students have with situations that are challenging and with concepts that appear inpenetrable. Many teachers have forgotten what it is like to be a learner — what it is like to be frustrated and what it is like to be successful.

Creating a program which provides frustration for its participants is a perilous undertaking. However, as a result of the environment in the LP, not one participant has yet left the program. An important reason for this is the presence of the lead teachers (see Section 2.C). One of their important roles is to identify participants who are experiencing difficulties, quickly provide additional assistance and counseling, and, where appropriate, refer them for further assistance and encouragement to the Program Directors (the authors).

Another strategy is that we enable participants to recognize that their frustration is an entirely valid component of problem solving. For many K–8 teachers, solving mathematical problems means memorize the rule, then apply it; often they are surprised to find that the problem-solving process is very different. While solving problems, they often experience negative emotions such as fear, frustration, uncertainty, and anger. On the third day of the program, participants are involved in a workshop on problem solving which deals explicitly with the role of affect in problem solving.[8] This workshop enables them to reflect on these emotions, recognize that they are normal, and apply problem-solving strategies to work through them to complete the problem. In order for teachers to model productive problem-solving behaviors in their classroom, they need to have a clear understanding of the

E. Empowering the participants. In addition to offering mathematical challenges to the participants and challenging them to bring LP materials to their classrooms, curricula, and colleagues, the LP strives to empower the participants to meet these educational challenges. As noted above, an important component of this empowerment is that participants are encouraged to see themselves as mathematicians when they are engaged in problem solving activities in mathematics. They are also encouraged to play leadership roles in introducing discrete mathematics into American classrooms. We describe them as experts in K–12 discrete mathematics, since in fact the teachers who have completed the LP are among the first teachers who have incorporated discrete mathematics in their classrooms, and are a substantial percentage of teachers who have done so. Their expertise becomes clear to them at the end of the first summer program when they are asked to reflect on what they have learned in their two-week encounter with the LP; they are amazed by what they have learned, by the amount they have learned, and by the fact that they have succeeded in learning mathematics which they never dreamed existed. Challenging teachers to learn mathematics has great risks, but there are also great rewards, because they learn that they can do mathematics, and pass on that sense of empowerment to their students.

The LP also empowers teachers to initiate mathematical explorations in their classrooms. Explorations imply that the class may travel to uncharted territory, where the teacher may not know what questions to ask, and what answers to give to students' questions. Because teachers are accustomed to be in a position of authority, they may be reluctant to ask any question whose answer they do not already know. We empower the teacher to overcome this barrier by modeling — showing the teacher that the workshop leader does not know the answers to all the questions — and by introducing mathematical questions to which no one knows the answer. In order for teachers to entertain questions whose answers they don't know, they have to become comfortable responding "I don't know the answer to that question; let me think about it overnight". This kind of response also lets the students know that there are problems which cannot be solved quickly, that some problems require thought and time. We often complain that students are unwilling to work on problems which require more than ten minutes (or even ten seconds) of their time. However, they don't see adults spending time on problem-solving. For the teacher to say "let me think about that one" sends a message that problems whose solutions are not obvious are worth considering.

F. Reflections. It should be noted that we have incorporated into the above discussion the lessons we have learned from directing the LP. Participants in the early institutes will recognize only some of these themes. As the program evolved, we learned what features we could incorporate into the program and how to make these features more meaningful to teachers. As we began to understand the power of discrete mathematics to facilitate change,

we were able to introduce deliberately and intentionally various components of the program.

Many of these features can be introduced in programs that address other content areas of mathematics. However, the qualities of discrete mathematics make possible the inclusion of all of these components of education reform. Moreover, repeating an important point introduced in the 1991 "concept paper" (see Section 1.C), discrete mathematics offers a new start for teachers because they can incorporate these features into their discrete mathematics lessons, where they are not restricted by existing curriculum requirements, and once successful adapt them into other mathematics instruction.

3. The LP as a continuing resource

Discrete Mathematics Newsletter. The newsletter *In Discrete Mathematics: Introducing Discrete Mathematics in the Classroom* has been published for the past six years, and includes articles written by participants in the LP describing their classroom experiences with discrete mathematics. The newsletter is distributed at no charge to over 3000 teachers. Its founding editor was Joseph G. Rosenstein, and it has since been edited by Deborah S. Franzblau and Robert A. Hochberg. The publication of the newsletter has been funded by DIMACS and NSF.

Workshops in Your District. For the past four years, the LP has offered to send a team of experienced teachers to any district to conduct workshops in discrete mathematics for middle and high school teachers. Beginning in 1998, similar workshops will also be available for elementary school teachers. The contents of the high school and middle school workshops were developed at summer workshops (called "workshop workshops") during 1993 and 1994, and parallel workshops for elementary schools are currently being developed. In the workshop workshop, teachers who are experienced with the use of discrete mathematics in their own classrooms develop a series of one-and-a-half hour workshops, and receive training on how to deliver similar workshops. In the typical case of "Workshops in Your District", two teachers go to a district and together present four of these workshops during an inservice day. The district pays only the honoraria for the presenters and the cost of the materials; the publicity and administration of the "Workshops in Your District" project are paid for by DIMACS.

The Franchise Program. Beginning in the summer of 1998, it will be possible for any college teacher with a background in discrete mathematics to replicate locally the Phase III program in discrete mathematics for K–8

will be available, in print or electronic format, by 1999. These materials will be targeted to teachers who have no experience with discrete mathematics, and will be based on the experiences of the LP program. In addition, the LP Web site (accessed from http://dimacs.rutgers.edu), currently under development, will contain materials from the LP and resources developed by LP participants.

Conference Presentations. Participants of the LP regularly make presentations on discrete mathematics at NCTM regional and national conferences and at conferences of local and state organizations of mathematics teachers. (Some have even made presentations at international conferences on mathematics education.) These teachers serve as an ongoing resource for conference organizers throughout the country who wish to schedule sessions on discrete mathematics.[9]

4. Participants' Statements

We conclude this article with a number of statements prepared by participants in the *Leadership Program in Discrete Mathematics* in response to the simple question, "What has resulted from your participation in the LP?" Taken together, these statements (presented alphabetically) illustrate and highlight many of the features of the LP that were presented in this article.

As a result of my participation over the years in the LP, I have introduced my high school to discrete mathematics. Aided by the staff at Rutgers University, I have been successful in implementing a full year discrete math course. Also, discrete math is now an integral part of a topics in math course. I have also spread the word about discrete math by making presentations at conferences on the local, state, regional, and national level.

William Bowdish, LP '92, teaches in the Sharon (MA) High School.

I am a teacher and mathematics supervisor in one of five high schools in a regional district. Since the summer of 1991, at least two department meetings a year in my school have been solely dedicated to a presentation on a discrete math topic and encouraging teachers to infuse this material in their classes. Discrete math is now offered as a full year course in all five high schools and enrollment appears to be growing in the larger schools. Textbooks have also been selected with an eye toward how much discrete math they contain. Since the summer of 1992, when I participated in the Workshop Workshop program and helped author some of the workshops, I have presented discrete math workshops to over ten schools or districts. I have also presented workshops in discrete math topics every year since 1992 at the AMTNJ conference and every other year at the "Good Ideas in Teaching

[9]Another important resource for conference organizers are the teachers who participated in the "Implementation of the NCTM Standard in Discrete Mathematics Project" directed by Margaret Kenney of Boston College; during each of 1993-1996, summer institutes for high school teachers were conducted at six sites throughout the country.

Pre-Calculus and ... " conference at Rutgers. One of the most gratifying experiences I have had is coming back to follow-up sessions and hearing or seeing how some of the participants have used one of the workshops that I had presented as a lead teacher, and usually how they have improved upon it. I haven't even begun to describe how much I have learned about a topic I had no prior knowledge about before the summer of '91. My image of myself as a mathematician was first formed that summer. In my wildest dreams, I could not have imagined the personal, professional, and academic growth I have achieved. I have enjoyed learning and sharing this knowledge and enthusiasm for mathematics with my students, with the teachers I work with, and with the wonderful people I have met through the Leadership Program.

Ethel Breuche, LP '91, is a teacher and mathematics supervisor at the Freehold (NJ) High School.

The LPDM has made me a better teacher and has made my school a better place for mathematicians to grow and develop. As a result of my participation in 1993-1995 in summer programs and my ongoing conversations with colleagues through follow-ups and e-mail communication, I have been involved in the development of the AP Statistics Program and work as a consultant for the College Board. I have introduced the AP Statistics course with a unit on Discrete Mathematics and teach this course at my school. We have also introduced a year-long Discrete Mathematics course at the high school. On a more philosophical level, my students benefit from having a teacher who knows that there are many good approaches to problem solving, and often many good answers to the same problem.

Anne M. Carroll, LP '93, teaches at Kennett (PA) High School

The LP has enriched my ability to bring real world connections to my students. It has enhanced my professional portfolio by exposing me to cutting-edge mathematical thought and theory and providing me with a network of resources from both the educational and research communities. This exposure allows me to bring a new lease to the mathematical life in my classroom. My students look forward to exploring current situations with a mathematical eye and become empowered when they realize that they too can think and speak mathematically.

Carol Ann DiMauro, LP '92, is a consultant with the New York City Mathematics Project, Institute for Literacy Studies, Lehman College, CUNY.

Since participating in the LP in '95, I have seen how much discrete mathematics is already inherent in many of the text books and NCTM publica-

have participated in a variety of discrete math activities. Even at recess students solve problems and practice mathematical tasks by "playing" on specially developed discrete math activities that are painted in the school yard. If you're in a class of mine, you can't escape learning mathematics with a lot of discrete activities.

Suzanne Foley, LP '95, is K–8 Technology Coordinator for the Olney (PA) Cluster of districts.

My involvement in the LP has broadened my personal approach to math and problem-solving as well as given me new avenues to reach different math students in different ways. First, discrete math is a great forum for teaching problem-solving. Rich problems with open-ended solutions allow for many kids to dive in and explore. Students with different strengths and interests can use approaches that suit their learning styles or multiple intelligence strengths. Second, the LP gave me connections to interesting and dedicated people who are a rich resource for further growth and new applications that are age- and grade-level appropriate.

Charles G. Hennessey, LP '95, teaches middle school students at the Holy Trinity School in Washington, D.C.

Two of my 8th-grade students at opposite ends of the spectrum provide the best example of the impact of discrete mathematics on my classes. One student, in the "learning disabled" program, hates school. He has no interest in the regular curriculum. The other student is in the honors program. She loves the challenge of school and everything about it. Yet these two students with seemingly no common ground both *love* the discrete math problems. The first student may not try a traditional homework assignment, but will work all class and more finding circuits, paths, counting rabbits and more. The second student wants extra work in all areas and seems particularly interested in the various areas of discrete math we have covered. Two different students both touched in different ways by discrete mathematics.

Jeff Hoyle, LP '96, teaches in the Dartmouth (MA) Middle School.

When I first learned about discrete mathematics, it proved to be a wonderful vehicle to get slower general math and low level students involved. They loved the graph theory and fractals, as well as secret codes, fair division, and map coloring. These students loved these lessons because they were "fun" and gave them a great measure of success. I also had great success with these topics in my precalculus and calculus classes. There I was able to teach the same topics at a more mathematical and rigorous level.

has enriched my enthusiasm, knowledge, and joy in mathematics greatly. I have shared this experience with many hundreds of students, and now with at least 100 placed teachers in Stony Brook and NYU.

Elyse Magram, LP '90, teaches at Smithtown High School West in East Northport, NY.

I cannot think of another professional development experience that has changed my life so much. First, there is the material, but the program has gone far beyond that. I have a new set of colleagues, leaders with whom I can share ideas and get feedback, people who are *peers* as well as mentors and role models. I have gained self-confidence. I have grown from a teacher who attends conferences to a teacher who attends conferences as both a participant and a presenter. I was nervous at first, but now I really enjoy this experience. It all started with the LP! I hope I can give back as much as I have gained!

Judy Nesbit, LP '94, teaches at the Montclair (NJ) Kimberley Academy; she was recognized as NJ Non-Public School Teacher of the Year 1996.

The Leadership Program allowed me to bring discrete math to deaf education. It was there that I saw how visual discrete math could be — graphic representations of fractals, graph coloring, maps, paths and circuits, etc. I thought **this** is something **for** deaf kids — a visual representation of math concepts that translates well to American Sign Language, and can transfer logically to numerical or symbolic representation. I immediately incorporated it into my teaching with positive results! Since the time that I spent with the LP, I have earned my Ph.D. in Deaf Education with an emphasis in mathematics. I can honestly say that the LP has influenced this decision. Now, as a teacher educator, I regularly include the topics, materials, and activities I learned at the LP into my instruction, and include discrete math concepts, such as Fibonacci numbers and the Sierpinski Triangle, into math and logic "puzzles" which I co-create with a colleague as a regular feature in a publication for deaf and hard of hearing students ("World Around You"). I conclude by saying that if it were not for the LP in discrete math, I certainly would not be doing what I am doing today!

Claudia Pagliaro, LP '92, is an assistant professor at the University of Pittsburgh in the Education of Deaf and Hard of Hearing Students Program, Department of Instruction and Learning.

Beyond the large and new personal opportunities that have come to my life as a result of my participation in the LP, including a position as a staff and curriculum developer; the chance to be published; the oppor-

of what mathematics is as a living breathing discipline in our contemporary world; what it can do and what it is that a mathematician does. I have been fortunate to have met and interacted with mathematicians and researchers, and very important to me — women mathematicians — who have enabled me to feel the world of mathematics as more accessible than it ever could have been. This in turn I have been able to communicate to other teachers and to students, so that they can feel included in the world of mathematics rather than intimidated by it.

Susan Picker, LP '90, is a mathematics instruction specialist for the Office of the Superintendent of Manhattan (NY) High Schools.

I have been very excited to learn and present this relatively new area of mathematics — "discrete mathematics." Since attending the LP last summer, I have found examples of discrete math topics in many places — at workshops and in textbooks, etc. As I have worked with teachers and students this past year, I have developed a clearer understanding of how to explain this sometimes confusing area of math. I think the institute gave me this clarity of understanding and presentation. Although I have used my training this school year, I hope to use it even more as I offer a series of after-school problem-solving workshops for teachers, using LP activities, in my school district next school year.

Nancy Shields, LP '96, is a K–12 supervisor of mathematics at the Beeville (TX) Independent School District.

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