Questions raised by Quora readers, related to mathematics education, each followed by my response:

Why don't we teach discrete math or linear algebra as much as pre-calculus? Alternative routes in a math education can be more conducive to computer and statistics related careers, so why don't we offer those as early?

Calculus is not for everyone. Therefore, precalculus and even Algebra II are not for everyone. Most people never need to know how to find 64 to the 2/3 power. Yet our high school math curricula are designed as if all students need such topics.

Topics like statistics and discrete mathematics are much more valuable for the average student, the former because it will help them understand how data is used and misused in their world, and the latter because it will show them the many ways that mathematics is used to model everyday situations and how reasoning and problem solving are used in both mathematics and daily life.

I have published a book called "Problem Solving and Reasoning with Discrete Mathematics" which is described at Problem Solving and Reasoning with Discrete Mathematics — it is intended for use as a textbook in high school courses, and also for the mathematically curious of all ages.

How do university professors determine marks for a question in the mathematics quiz/exam?

Your question raises many issues. One of the purposes of an exam is to help the instructor distinguish between the students who have a good understanding of the material from those who do not and to distinguish between those who a fair understanding of the material from those who do not.

If an instructor gives an exam that is too hard or too easy, then it will not be possible for the instructor to make those distinctions. If that happens, then the students also will not be able to assess their understanding of the material and will have no clue how they might improve their grades.

An important issue not discussed here is that students are in general unwilling to seek their instructor's assistance to help improve their understanding of the material. The ancient statement "one who is too embarrassed to ask a question is not able to learn" is as true now as it was 2000 years ago.

How do you create an exam where the median score is 70–75 out of 100? You use questions that are essentially similar to questions discussed in class and to problems that were on the homework. Those students who have understood the basics of your course should be able to answer most of those questions correctly.

But there should also be several questions on the test that are somewhat like what the students have seen before, but are just different enough that the students would require a deeper understanding of the material in order to answer them correctly.

With a test like this, those with a fair understanding of the material will score in the 60s and 70s, those with a good understanding will score in the 80s, and the top students will score in the 90s. Of course, it never works out exactly that way.

I tended not to use questions that had multiple parts each of which depended on previous parts, because someone with a wrong answer on the first part would often be unable to show you what they know, because they were blocked by their incorrect answer to the first part. So I I tended to construct exams with many questions (perhaps 15) each with about 5 or 6 points. Such questions are usually easy to grade.

Sometimes that’s impossible, and you need to ask questions that require students to integrate their understanding of a topic, like drawing the graph of a particular function. In that case, the instructor has to be careful to enable the students to show what they know, rather than getting stuck on a point that they don’t know so well, and thereby getting the whole question wrong.

This seems like a good place to stop.

Do mathematicians and professors really hoard Hagoromo chalk?

Actually, I’ve been a math professor for over 50 years and I had never heard of Hagoromo chalk until I saw this question and looked it up. I’ve been retired for two years and perhaps Hagoromo hoarding is a new phenomenon, but I don’t think so since the company has been around for a long time and has apparently just recently stopped producing the chalk … hence the hoarding.

During the last few years that I taught mathematics, I really didn’t use chalk. I found a much better device … the document camera. For those of you who don’t know what it is, it’s simply a camera under which you place a sheet of paper; the camera is connected to a projector that transmits what is on the sheet of paper onto a screen. Instead of writing on a board, I write on sheets of paper and magically what I write appears on the screen.

Why am I telling you this? If you are a teacher, or are planning to become a teacher, I encourage you to use this device if you have access to it.

There are several important advantages over chalk. First, you preserve what you write on sheets of paper so that if someone wants to ask a question about what you wrote earlier in the class, you and the class can look at the “chalk board” and discuss the question. Moreover, after the class you can scan all of the pages that you wrote and provide them to the class, so that students who are absent can see what was done in class and all students can review the material. Also, since the students don’t have to write down everything that you wrote on the board, they can pay greater attention to what was said in class than focusing on quickly writing down everything that is on the board and not hearing whatever explanation was provided.

The greatest advantage, however, of the document camera is that you, the teacher, can face the class all the time, instead of standing with your back to the class and your face to the blackboard. That makes it possible to create a dynamic where you and the class are talking to one another … instead of you talking to the board.

So, who cares about fancy chalk? Mathematics teachers are hoarding document cameras; I have two of them but, at the moment, since I'm retired, I don't know where they are.

What is the trickiest order of operations problem you've ever seen in mathematics?

Personally, I believe that any arithmetic (or algebraic) expression should be written with enough parentheses so that the order of operations is unambiguous. Even with simple problems, like 2 + 3 x 4, one should either write (2 + 3) x 4 — in which case the answer is 20 — or one should write 2 + (3 x 4) — in which case the answer is 14. I think that teaching children rules for disambiguating expressions — like 2 + 3 x 4 — is less useful than teaching them to write unambiguous expressions.

As a Mathematics PhD, would you recommend someone that is very passionate about mathematics but has no natural talent for it? Someone with natural talent for something completely unrelated that they don't care about to pursue Mathematics as a major.

Being passionate about mathematics does not necessarily mean that you should pursue a career in research mathematics. There are many people who are passionate about history or music or basketball who do not become professional historians or professional musicians or professional basketball players. They generally earn a living some other way, perhaps related their passion, perhaps not, but still find ways to pursue their passion.

What you call "natural talent" does not guarantee success — in mathematics, or in any other arena. Passion, determination, hard work, relationships, and even luck, all play significant roles in whether one will achieve their goals.

You may feel that you are less talented than some of your classmates, but your extra passion and determination may in the long run be more significant than their talent — you may be able to develop your "talent" but they may not be able to grow their passion. You may find that you have more talent than you are giving yourself credit for.

Don't make an either/or decision too quickly. Keep pursuing your passion for mathematics and keep available a career path for which you appear to have talent. Perhaps you will be able to hold on to both in your life's journey. Good luck.

During your Mathematics PhD program, how much time did you spend with your advisor and what did they teach you?

Even if I remembered, 50 years later, how much time my advisor spent with me and what he taught me, those answers would not really be of any use to you.

Your advisor should spend with you the time that you need, and should help you to develop the skills and insight that you will need to accomplish your task — that is, to complete an appropriate piece of research and translate what you have discovered onto paper.

What you need will depend on you, your advisor, and your goal — so it is hard to make a general response to your questions … but here are some modest attempts to give you some useful advice.

Your advisor should provide you with a goal (that may, for example, be a specific problem to solve) and should help guide you on a path that may lead to that goal. (You may certainly have input into what the goal is.) That may mean showing you publications that may be relevant to your goal and helping you understand why they are relevant to your goal.

Your goal should be something that your advisor has thought about previously, and has determined is a reasonable goal for a thesis. That is, your advisor should have a sense that even if the goal is not achieved, your efforts should be sufficient to lead to a satisfactory set of conclusions, even if they don't achieve the goal. Your advisor should know you well enough to be able to set a goal that is appropriate for you.

Your advisor should have the time to meet with you. But you will have to take the initiative to ensure that your advisor meets with you. You will need to speak with your advisor regularly, indicate what you have done so far, ask the questions that you have, share your frustrations [Note: frustration is common to all who write Ph.D. dissertations in mathematics], and seek guidance for what you might do next.

It is probably wise to determine early in the game whether you and your advisor are compatible on both a personal and instructional level, whether your advisor is interested in the goal that is being set for you, whether your advisor is willing to spend with you the time you need, and more generally whether your advisor considers working with you, and helping you complete your doctorate, a priority.

You don't want to be in the position, two years later, that after having invested much time in working toward your goal, you have made no real progress.

So you need to choose an advisor carefully. I will avoid making comparisons to choosing a life partner, but the relationship between advisor and Ph.D. student is indeed a relationship. If you believe that you have chosen the wrong advisor, one who is not providing you sufficient time and sufficient support, then a friendly separation may be best.

I should note that my comments are not based on my experience as a graduate student — I have only warm feelings toward my advisors (I had two of them, for various reasons) — but on what I have learned from the experiences of students in various Ph.D. programs in mathematics over many years.

Are Finite Mathematics and discrete mathematics the same?

They are not the same. Discrete mathematics deals with situations that involve countably infinite sets, like the natural numbers N, as well as with finite sets. If you are asked to visualize a number line, you may think of the line that is the x-axis, where the points on the number line are continuous, without any gaps. On the other hand, you may think of the number line whose points are natural numbers (or integers); this line is not continuous, since it has huge gaps between its elements. That is what makes it "discrete," with each element separate from all the others.

There is no specific definition of what "discrete mathematics" contains and, as a result, there are many views of what discrete mathematics includes. However, there is general agreement that discrete mathematics includes graph theory (where graphs consist of vertices and edges), combinatorics, iteration and recursion, finite probability, introductory set theory, elements of number theory, codes and cryptography, sequences of numbers, and other topics — including the very relevant topics of voting, apportionment, and dividing a state into legislative districts.

A good elementary introduction to discrete mathematics can be found in my book "Problem Solving and Reasoning with Discrete Mathematics" that is available at Problem Solving and Reasoning with Discrete Mathematics

"Finite mathematics" is a term that was used years ago, before computers and computing came into being. Once that happened, many problems that were previously considered unattainable could now be addressed by computation, and the new area of "computer science" required many topics of discrete mathematics as prerequisites.

I have argued that for most high school students — those who will never use calculus — topics in discrete mathematics are more valuable than the sequence of Algebra 2, Precalculus, Calculus that is currently prescribed by the national standards in the United States. Indeed, I wrote the book mentioned above (at new-math-text.com) to provide a text that would introduce high school students to discrete mathematics. I also believe that discrete mathematics is a much more useful set of topics for introducing reasoning and problem-solving into the school curriculum.

What open problems in mathematics can be fully understood by undergraduates? And which of these problems could theoretically be solved by undergraduates?

If you are an undergraduate in the U.S. who is interested in mathematical research, I suggest that you look for an REU (Research Experiences for Undergraduates) program that you can participate in this summer. REU programs exist at many universities.

In an REU program, you will have a mentor with whom you can work on research problems that can be solved (at least partially) by undergraduates, not like the problems that were suggested by the other respondents.

In some areas of mathematics, like graph theory, there are lots of unsolved problems that are accessible to undergraduates, but you need a mentor who can judge whether a particular problem is likely to be solved by an undergraduate.

I also suggest that you talk to faculty members at your own college and see whether one of them would be willing to work with you on some research problems.

Going it alone is probably not realistic.

What are some good books to learn discrete Mathematics?

An excellent book for beginners - it was written with beginners in mind, and it focuses on developing reasoning and problem solving skills - is my own book, "Problem Solving and Reasoning with Discrete Mathematics", at new-math-text.com

If you could design the ideal K-12 mathematics curriculum, what would it be?

What is the ideal math curriculum?

This is not really the right question to ask, unless you are thinking about the appropriate curriculum for a specific person, since, as Alon Amit has already pointed out, the ideal math curriculum, if it exists, depends on the person.

It also depends on the "delivery system" — that is, do the teachers have the appropriate background, are they provided the appropriate professional development, are they regarded as professionals by the school and the parents, do they have the freedom to run their own classrooms or are they constrained to ""teach to a test," etc.

It also depends on "student preparedness" — by which most people mean "did the students learn the prerequisites" but which also means, particularly for urban and rural students in the US, can they see the board, can they hear the teacher, are they suffering from asthma or toothache, did they have a nourishing breakfast, are they the victims of bullying by their peers or abuse by their parents.

During the 1990's, many states in the US created "math standards" which were intended to describe what **all** students needed to know, understand, and be able to do. Standards were not considered to be a curriculum, but to describe the content that should be included in every curriculum. Nor were standards intended to be developed into an ideal curriculum, but only what was needed to prepare every student for college, careers, and citizenship. Depending on the direction that the student took, he or she might well need to learn additional topics in mathematics.

One direction that the student might take is preparation for calculus, particularly if the student intended to pursue a career involving the mathematical sciences, the physical sciences, and engineering. But not all students need to follow that direction.

I led the effort in developing math standards (and a Mathematics Curriculum Framework) for New Jersey, which were adopted by the state in 1996, and a revised version adopted in 2002. Perhaps in part due to those standards, New Jersey consistently placed among the top states on the NAEP assessments, despite having one of the most diverse student populations.

The national standards adopted almost ten years ago, instead of focusing on preparing all students for careers, college, and citizenship, focused on preparing all students for calculus. Thus they want all students to learn the topics that only some students need to learn. Not all students, for example, need to be able to find 64 to the two-thirds power. Sadly, the goals of these standards are based on false assumptions. Moreover, the standards are linked to national assessments that have instilled fear in school administrators who, as a result, have stifled teachers' creativity in instruction. This is a recipe for disaster.

Disclaimer: I am an advocate for including discrete mathematics in the K to 12 curriculum. This includes systematic listing and counting, which Alon Amit mentions, but also other topics, including vertex-edge graphs and their applications (for example, which route should a delivery person use in order to minimize the total distance traveled while delivering all the packages?). These topics have few prerequisites, but are ideal for introducing students, even young children, to problem solving, reasoning, and modeling. My textbook for high school students (and anyone who is mathematically curious) is called “"Problem Solving and Reasoning with Discrete Mathematics” and can be found at new-math-text.com