Questions raised by Quora readers, related to general mathematical questions, each followed by my response:

If math and logic are eternal, immutable and transcendent truths, where did they reside before the Big Bang?

They haven't moved! Wherever those immutable, eternal, and transcendent truths resided before, they are still there, and, since they are in the same place now as before, they were in the same place before as they are now. Big Bang theories come and go, but immutable truths live on … although because of entropy the neighborhood sometimes deteriorates.

Why is Platonism the most accepted philosophical position among modern mathematicians?

I would say that very few modern mathematicians have ever thought about whether they are Platonists or not, except for those who have a particular interest in philosophy. In my 50+ years as a mathematician, I don't remember ever having a discussion with fellow mathematicians about whether we are or are not Platonists. It's just not an issue.

Two questions that are sometimes discussed are (a) whether mathematicians "discover" new theorems or "create" new theorems and (b) whether the things that we imagine, including numbers, really exist, or are only figments of our imagination.

I remember, as a beginning instructor in 1966, asking my class of liberal arts students whether they had even seen a "three" — or course they all laughed because they had seen "3" since their childhood. However, all they had seen were pictures of three, but not a real three, comparable to the real giraffe they saw in a zoo.

But even these two questions, I would venture to say, are not taken very seriously by most mathematicians.

Mathematicians and math majors, what was your personal experience of first learning how to read and write proofs?

In the fall of 1957, I was a freshman at Columbia and enrolled in an honors course in analytic geometry and calculus taught by Prof. Walter Strodt. As a high school student I had taken and earned 100% in all the New York Regents math exams, including an exam on solid geometry — no one took calculus in high school in those days — but all of that did not prepare me for the first day of Prof. Strodt's class where he talked about the distance formula in n dimensions. I vividly remember going back to my dormitory room and trying (and failing) to arrange pencils in four dimensions — that is, trying to construct a concrete mental image of what four dimensions could be. What I was not prepared for was the level of abstraction that is needed for a person who wants to do advanced mathematics. I can't report on my first experience with reading and writing proofs, but I'm sure that part of it was letting go of the concrete and literal meaning of the words, and allowing yourself to enter a world of abstraction, where our usual conventions need not be followed. For example, many of my students who were junior level math majors had a hard time understanding that mathematicians can pluck 5 out of its usual order and place it at the end of the sequence of the natural numbers. In concrete reality that makes no sense, but in the abstract world of mathematics, it is not a problem.

Do you regret going to Rutgers University?

Most of the answers to your question are from students who attended Rutgers and had a variety of perspectives.

If you are thinking about going to Rutgers (or any major university) you should know that you will be presented with a wealth of opportunities, but they will be useless to you unless you are a person who takes charge of your own education, who takes advantage of the opportunities that are available.

I was a math professor at Rutgers for 48 years, until I retired two years ago. I have no regrets about being at Rutgers or about teaching at Rutgers. I am very happy about my choice to come to Rutgers.

The only regret I have is that most of the students in my classes never took advantage of the opportunity that they had to come to my office hours and ask me to help them better understand the material in their courses.

So wherever you go, talk to your professors. Some will not be interested, but most will be eager to speak to you about the course material.

Is the "Null Set" an imaginary concept promoted by atheists?

What a wonderful question! Is the null set an imaginary concept promoted by atheists?

Actually, I think you've got it backwards! I will argue that the null set is a real concept promoted by non-atheists!

And in doing so, I will probably freak out both the people who follow me for my mathematical answers and the people who follow me for my theological answers.

I have noticed, from my years of teaching, as well as from observing Quora questions that people have a real problem grasping the existence and the uniqueness of the empty set. Evidently it is very hard for many people to understand that there can be a set with no elements, that the empty set exists, and it is also very difficult for many people to understand that there is only one set with no elements, that the empty set is unique — that the set of apples on my table (there aren't any) is the same as the set of even primes greater than 2 and that is the same as the set of remaining survivors of the Civil War in the United States.

The existence and uniqueness of the empty set is puzzling, except to mathematicians. Similarly, the existence and uniqueness of the higher power that many call God is puzzling, except to those who grasp that the world, despite its randomness, has a certain fundamental coherence.

Moreover, mathematicians know that all of mathematics can be based on this mysterious empty set. The empty set itself can be defined to be 0, the set whose only element is the empty set can be defined to be 1, the set whose only elements are 0 (the empty set) and 1 (the set whose only element is the empty set) can be defined to be 2, etc. Then the set whose only elements are 0, 1, 2, 3, … can be defined to be N, the set of natural numbers, and the set whose elements are all subsets of N can be defined to be the real numbers, and so on. Thus all of mathematics can be built out of this mysterious null set — all of mathematics comes from … nothing.

This parallels the notion in Judaism (and in other religions) of "yesh mei-ayin — all that exists emerges from nothing" — that somehow the entire world was created out of nothingness, just as all of mathematics is created out of the empty set. The term used in Latin is "creatio ex nihilo."

We take this a step further. The Hebrew word "ayin — nothingness" is, from the Kabbalistic perspective, a name of God. Most people find it difficult to conceive of a god that is not like ourselves, that is not in our own image; so we imagine super beings that are somehow like us, but also have super powers.

Most people need physical representations of their God or gods. Most religions, although they may have images of the divine, insist that God is beyond any such images. The solution of Christianity, particularly Catholicism, is to have a God that has a human component. Maimonides, the prominent 12′th century Jewish philosopher, insisted that all the anthropomorphisms of the Bible — all of the verses that spoke of God in human terms — must be understood as metaphors, since God must encompass and yet be beyond all those metaphors. Thus, when a modern rabbi was told by a visitor that he doesn't believe in God, he responded that, "the God that you don't believe in, I don't believe in either." The most radical position of denying that God is somehow like us is that God is "ayin — nothing," that is, God is "no thing", that God is beyond thing-ness.

The difficulty that many have in grasping that God is no thing is parallel to the difficulty that people have in grasping the null set, a set that has no thing in it.

So, the notion of the empty set is real and it seems to be parallel to theistic notions of creation.

That is the end of my response, but for those who are into this notion of "ayin," I will offer the following interpretation of the familiar Biblical verse from Psalms 121:1, "I raise my eyes to the mountains, mei-ayin will my help come. My help will come from Adonai, creator of heaven and earth." The word "mei-ayin" has two meanings – "from where" and "from no-thing." The usual translation of "mei-ayin" in this verse is "from where," but the traditional translation is problematic because it implies that to receive God's help one needs to look to the heavens. The idea that God's residence is in the heavens is a bit problematic, since God is everywhere. So some use the alternate translation, "from no-thing," since, as noted above, "ayin" is one of the names of God, at least it is in Kabbalistic writings. God is a concept that we cannot fully grasp, an elusive reality that is beyond our reach, a no-thing that encompasses every-thing. From this perspective, “mei’ayin yavo ezri” is not a question “From where will my help come?,” but an assertion “my help will come from no-thing, but from the unseen God, the creator of heaven and earth.”

What makes probability theory counter-intuitive and difficult for many to grasp?

One important aspect of probability that many people find counter-intuitive is the notion of independent events.

In many areas of life, whatever you do has consequences; indeed, part of raising children is teaching them that there are consequences, sometimes positive and sometimes negative, of what they and others do.

Thus it becomes intuitively true that the outcome when dice are rolled should depend on what happened before. When you toss three coins, every accepts that the probability of getting three heads is small. So when you toss coins and get two heads and then toss a third coin, the probability of getting a third head should be just as small.

It is counter-intuitive that what happens with the third coin is not affected whatsoever by what happened to the first two coins, that the probability of getting a third head remains exactly one-half.

The lack of understanding of this principle is reflected, for example, in discussions of the lottery when people say “the number 15 is due to come up — it hasn’t come up for 10 days” or in betting on red in roulette because the last three times it has been black.

What is your favorite “impossible proof”? For example, the proof that "one is equal to two"? How did you learn the proof? What is the fanciest impossible proof you’ve seen (especially if it is not for 1=2)?

Here is a proof that all horses have the same color. We prove this by proving that “all horses in a set of n horses have the same color” using induction on n.

For the basis step, where n=1, in a set of one horse, every horse has the same color by default.

For the induction step, we assume that “all horses in a set of n horses have the same color” and we need to show that this statement is true for n+1 as well. So take a set of n+1 horses {H_1, H_2, H_3, … , H_n, H_(n+1)}. Then, by the induction hypothesis, the first n horses {H_1, H_2, H_3, … , H_n} all have the same color C. Also, by the induction hypothesis, the last n horses {H_2, H_3, … , H_n, H_(n+1)} all have the same color C’. Since H_2, H_3, … , H_n are in both sets, C and C’ must be the same color. Hence we have shown that “all horses in a set of n+1 horses have the same color.”

It follows by the Principle of Mathematical Induction that all horses have the same color.

How can I determine if a five digit number is prime or composite?

A good technique to help yourself fall asleep is to take a whole number and try to determine if it is prime or composite in your head, with your eyes closed, and, if it is composite, what are its factors. Since there are well-known techniques for determining if a number is divisible by 2, 3, or 5, you want to start with a number that is not divisible by those three primes.

If you start with a three digit number, to determine whether it is a prime, you only have to check to see if it is divisible by the prime numbers up to 31. With a little practice you can do that in your head.

Whatever is distracting you from sleeping is dissipated by focusing on this task, and you should soon fall asleep.

Once you are able to easily determine whether a three digit number is prime, you can then more on to four-digit numbers, starting with those that begin with 1, 2, or 3 and eventually moving on to those that start with 7, 8, or 9. You only have to check to see if the number you chose is divisible by the prime numbers up to 97 (or 53 if the number begins with 2).

That will occupy you for a lifetime, so you will never need to deal with five-digit numbers.

Try it first, and if you find that you fall asleep, then you can upvote my prescription.

Mathematicians: What is so special about expressing 33 as the sum of three cubes? Is there any significance to the problem being solved?

There is no intrinsic significance to expressing 33 as the sum of three cubes.

When I read your question, I figured out that 1+8+27 is 36; that’s close to 33. To get 33, you have to use negative cubes because all the other positive cubes are greater than 33, so I figured out that 5^3 + (-4)^3 + (-3)^3 = 125 - 64 - 27 = 34; that’s close to 33 also. So I figured that it might actually be possible to find three cubes of integers whose sum is 33 … or not. But I didn’t think about the problem any more because the problem didn’t really interest me that much. This mathematician didn’t find it worth his time to think more about it. Hence my response that the issue is not intrinsically significant.

But then I noticed the link in your question and I read the article. And all of a sudden the question became wow! significant. There really are three cubes whose sum is 33! It is surprising that the smallest cubes that add up to 33 are cubes of 16 digit numbers. It is amazing that Andrew Booker devoted a lot of time working on this problem without knowing what the outcome would be; he created new algorithms and eventually (perhaps to his own surprise) found those numbers. And, as pointed out in the article, his method of solution might shed light on other significant problems in number theory. Who knows where this may lead?

So maybe this problem is not just an odd result, one whose significance is just in the eye of the beholder, of interest only to a small group of mathematicians, but one that really does have intrinsic significance.

Why are some theorems that look obviously true yet are so difficult to prove?

Looks can be deceiving.

What makes a statement "look obviously true"?

Perhaps it's a statement about whole numbers — like the Goldbach Conjecture or the Collatz Conjecture — and you have examined many cases and found the statement to be correct in those case and you would like to conclude that it's always true. Sorry, you can't jump from a few cases (no matter how many the few) to a universal conclusion.

That means that "looks obviously true" perhaps means "can't find a counterexample." But there may be one! So perhaps you should understand "looks obviously true" as "haven't yet found a counterexample." That's a more modest statement, and a more appropriate one.

One should be careful about claiming that a statement is obviously true. It may be true … or it may be false. If true, it may be obviously true … or it may not be so obvious. If true, it may be easily proven … or it might be quite challenging to prove it.

When did you realize you wanted to become a mathematician?

I imagine that every person has a different story to tell of how their life turned out the way it did, and every mathematician has a different story of how they became a mathematician.

It certainly has to start with both a love of mathematics and a talent for mathematics — as a high school student, I read every book about math that I could find and I scored very well on all my New York State Regents exams — but not everyone with that love and talent becomes a mathematician, as many people on Quora can testify.

Some leave because they also have a love for and a talent in some other area, some leave because they don't want an academic career (perhaps unaware that many mathematicians are otherwise employed), some leave because of financial considerations, some leave because venturing into the unknown (working on hard problems) is too stressful, etc.

Personally, I don't remember ever deciding that I wanted to be a mathematician. But I do remember deciding that I could not be a physicist. As a college freshman in 1957, I was considering majoring in both math and physics. In the first semester of my sophomore year I took a course on Electricity and Magnetism with Polykarp Kusch, who had won a Nobel Prize. I somehow got an A in the course, but at the end of the course I realized that my way of understanding things was not at all like my esteemed physics professor, but was much more like my math professors. So I majored In math and then became a mathematician.

In part that was a result of not knowing that I had other choices. It seems to me that young people today are more aware of choices than their counterparts were two generations ago. In any case, I went to graduate school in mathematics and then had an academic career as a mathematician — teaching at Rutgers for 48 years — initially as a researcher and then shifting my focus to pre-college math education. I have no regrets about my life as a mathematician.

In mathematics, can it be proven that a statement is not provable, or proven that it is provable without proving it?

The simple answer is that there is no pattern on the basis of which you can tell whether an arbitrary conjecture is true.

Let's look at an example. Suppose you give me a polynomial p(x) and ask whether it has a real root — that is, a real number a for which p(a)=0. Can I determine whether it has a root without proving it?

Well, if the degree of the polynomial is odd, then I can say "yes" without even thinking about, but that's because I already know how to prove it. If the degree is 2, then I can play mentally with the coefficients for a few seconds, and give you a quick answer, but again it's because I know how to prove it. But if the polynomial has degree 10, then I would have no idea whether or not it has a real root.

Of course I could graph it with a calculator. If it appeared to have a root near 8, then I could calculate p(7) and p(9). If one is positive and the other negative, then I could conclude that it has a real root. But if there doesn't appear to be a real root, I can't conclude that it doesn't have one, because there may be a real root somewhere near 1,000,000. You might conjecture that it has no real root, but you certainly haven't proven it, and you may in fact be wrong.

A two-digit number is three less than seven times the sum of its digits. If the digits are reserved, the new number is 18 less than the original number. What is the original number?

This is a problem that is easily solved using algebra. If you set it up properly, you will get two linear equations in two variables - you have to figure out what the two variables stand for - and that will lead you to the correct answer of 53. However, you need to do such problems yourself, and not ask Quora to provide answers to your homework problems - you won't learn how to do the mathematics that way -although I'm sure that someone will post the complete solution.

What was it like to study under Serge Lang?

I didn't study mathematics under Serge Lang, but I had a math class that met just before his class. My professor often didn't end his class on time, and Lang once walked in and prevented my prof from finishing. So the next time, my prof locked the door. He went beyond his time and felt no pressure to finish since Lang couldn't stop him. However, Serge Lang climbed through the transom window that was above the door!

What is a good math method to quickly determine someone's age from a known birth date?

There is a secret and well-hidden method that mathematicians and other wizards use to determine people's ages. It's known as … shh … subtraction.

Do mathematicians ever argue about math?

Do mathematicians ever argue about math? Certainly there have been many examples of this. But you probably meant to ask whether mathematicians ever disagree - that is, whether they ever disagree about the answer to a question without necessarily getting into a fight about it.

Let me give you an example from the domain of vertex-edge graphs. A vertex-edge graph consists of a set of vertices and a set of edges each of which joins two different vertices. The degree of a vertex in such a graph is the number of edges that join that vertex to another vertex. An n-regular graph is one where each vertex has degree n.

Question: How many different 3-regular graphs are there with 8 vertices?

A few years ago I claimed that there were a certain number of different 3-regular graphs with 8 vertices, and a colleague of mine decided that the correct number was one less than I had claimed. That was certainly a disagreement between mathematicians.

Who was right? There were two possibilities. One was that I had found one graph that my colleague hadn't. The other was that two of my graphs were actually the same although they appeared to be different.

We exchanged a number of emails and eventually realized that two of my graphs were actually the same, so that my answer was wrong and hers was correct. Thus we agreed that her number was correct.

Doing mathematics, as noted by another responder, is very much a social activity, where the correct solution of a problem often emerges from the interaction of several people.

That's not the end of the story, however. When I proposed this question to a group of mathematically talented high school students that I direct (the Rutgers Young Scholars Program in Discrete Mathematics), one student discovered that two of the graphs that my colleague and I had agreed were different were actually the same. So the correct answer was one less than the number that we had previously agreed on!

Can you figure out the correct answer yourself?

Why do the number of prime numbers decrease within every rising decade?

Each year many primes get tired of the great responsibilities that come with being prime and decide that they would rather be composite.

What would the world have been without zero?

When the Russians hacked all of the world's computers simultaneously in 2022 and kidnapped the zeroes, all of technology stopped since it all depended on binary arithmetic, and now only 1s were left, so all code looked the same. Fortunately, we knew in advance where they were planning to hide all the world's zeroes, found them there within moments after they were taken, and returned them rapidly to their proper positions. Otherwise who knows what the consequences would have been! So we should be very grateful that the number "0" was found, and found quickly.

How can we MEMORISE cubes? 11-20. Any patterns that can be observed? A method by which we can answer immediately?

A more valuable skill than memorizing cubes is training oneself first to multiply any two two-digit numbers in one's head, and then to multiply any two digit number by a three-digit number in one's head.

Since any number between 11 and 20 is a two-digit number and its square is a three-digit number, you can then quickly find the cube of any number between 11 and 30 without any memorization.

There is a pattern in the sequence of cubes, just as there is a pattern in the sequence of squares and a pattern in the sequence of whole numbers.

The difference between successive whole numbers of course is 1, and the differences between the sequence of squares is … let's see …

The squares are 1, 4, 9 , 16, 25, 36, 49, 64, etc. and the differences are 3, 5, 7, 9, 11, 13, 15, … — that is the differences of squares is the sequence of odd numbers. The differences of the differences (called the "second differences") are 2, 2, 2, 2, 2, 2, … In other words the second differences of the squares is the constant sequence of 2s.

Bumping up to cubes, the sequence of cubes begins 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

The sequence of differences between successive cubes begins 7, 19, 37, 61, 91, 127, 169, 217, 271, …

The sequence of second differences begins 12, 18, 24, 30, 36, 42, 48, 54, …

Now we see the pattern — the second differences are increasing by 6, so that the third differences of the cubes is the constant sequence of 6s.

So you can memorize that "trick", but I still think that you are better off learning to multiply in your head by two-digit numbers.

How do I know if I have done a combinatoric counting problems, and combinatorics in general, right? I have done it wrong many times in the past and now I can't really trust my intuition or think it properly.

There are two important general questions embedded in this question:

(1) How do you develop confidence in your mathematical abilities?

(2) How do you know that your answers are correct?

Here are some thoughts on each of these questions:

(1) Probably the best way to develop confidence in your mathematical abilities is to work on and solve a lot of problems and learn that you have solved them correctly.

Now you have to understand that mathematics encompasses a wide variety of topics, and you may develop your expertise and confidence in some areas, but not in other areas. For example, although I have been a professor of mathematics for over 50 years, there are some topics in which I was never particularly interested, and probably forgot much of what I originally learned, so I am not confident that I can answer Quora questions about those topics.

There are other topics that I learned very well, and even wrote articles or books about them, but I have not looked at them for many years, and have lost the knowledge in those areas, and therefore the confidence that I can give knowledgeable answers to questions about them. “Use it or lose it” applies to mathematical abilities, just as it applies to physical abilities.

I have always felt that questions about combinatorics and probability are particularly challenging. When I first taught a course on probability, I was surprised that every problem in the book was different from every other problem, even though the problems sounded much the same. In other words, developing confidence in these areas may require you to solve correctly many more problems than in other areas.

If you are not feeling confident about your abilities in a certain area, you need to work on many problems in that area … and find a knowledgeable person who can explain to you why some of your solutions are incorrect. That may be the result of a small flaw in your reasoning or your understanding that you may not be able to catch on your own. A few words from an expert may reveal the flaw or flaws in your reasoning and understanding.

Remember also that confidence and expertise take time to develop. Taking one course in an area as a college junior or senior is usually insufficient. If you take several courses and teach a course in combinatorics, then along the way you will develop that confidence and expertise.

Why is teaching the material important? One learns at a deeper level when one has to explain one’s reasoning to others. It would be a good idea if you explained each of your solutions to one of your fellow students and try to convince him or her of the correctness of your reasoning.

(2) How do you know that your answers are correct? Even if you have confidence and expertise, your reasoning may be flawed, and in a complicated solution the flaw may be hard to find!

Here’s an example. I once wanted to determine how many non-isomorphic 3-regular graphs there are with 8 vertices. I came up with a proof that there were a certain number, call it k, such graphs. A colleague of mine worked on the same problem using a different strategy and came up with k-1 graphs.

Who was right? We were both confident and expert, but we couldn’t both be right. Either my colleague had missed a graph or I had two graphs that were isomorphic. We emailed back and forth for several days, and we figured out that it was I who was wrong, that two of my graphs were indeed isomorphic.

Actually, it turned out that we were both wrong. We later discovered that two of our agreed-upon graphs were isomorphic and that the correct answer was neither k nor k-1, but actually k-2!

This highlights the observation that finding the truth in mathematics is a communal activity. You can’t be sure that a complicated and long proof is correct until other mathematicians have agreed that it is correct. And even then, you can’t be absolutely sure because perhaps another pair of eyes will see a flaw that you overlooked.

When the Monty Hall Problem was proposed (Be sure to google the problem!!), many mathematicians lined up on both sides — many said that “change the door” was the correct strategy, and many said that “don’t change the door” was the correct strategy. Deciding which was correct took a while, but eventually everyone agreed on which answer was correct.

One colleague always introduces me as the person who found a flaw in his Ph.D. thesis. Fortunately, that flaw was (as I recall) easily corrected.

Mathematicians, even those who are expert, do sometimes make mistakes in their reasoning, and that is perfectly normal. After all, we are all human.

The first published proof of the Four Color Theorem was discovered to be incorrect after several years, and a correct proof was not constructed until a hundred years later. Even Fermat, who claimed to have a short proof of a certain theorem, was probably wrong, as most modern mathematicians believe. Although, who knows, perhaps one of you will find his short proof!

So mathematics is a social activity, involving conversations over generations and centuries, and mathematical truth, even when it is spelled with a capital "T", requires agreement among mathematicians.

What mathematical discovery were you most proud of?

In the years after my Ph.D., I published several articles that dealt with linear orderings (see definition and discussion below), and at some point I decided to write a book that incorporated everything known about linear orderings, including my own work and that of other people. Its title of course is "Linear Orderings" and it is a research monograph written in textbook form, so that it is accessible to graduate students and advanced undergraduates. It is available at no charge on the Internet. After it was published, I turned my attention to secondary and primary mathematics education, and that has been my focus for the last 30 years. I am most proud both of this book and of my work in math education.

Now, what is a linear ordering?

A relation R on a set A is called a "partial ordering" if it is (a) reflexive (<a,a> is in R for every a in A), (b) anti-symmetric (if <a,b> and <b,a> are both in R, then a=b), and (c) transitive (if <a,b> and <b,c> are both in R, then <a,c> is in R).

Partial orderings come up in a variety of ways in mathematics. For example, (1) the ordering of the real numbers, with the relation <a,b> is in R if a is less than or equal to b, is a partial ordering, (2) the ordering of the natural numbers N by divisibility (<a,b> is in R is a is a factor of b) is a partial ordering, (3) the ordering of the power set P(A) by inclusion (<a,b> is in R if a is a subset of b) is a partial ordering, (4) the set of tasks that you have to perform before leaving your house in the morning, with the relation <a,b> is in R if task a must be performed before task b, is a partial ordering.

All four of these relations satisfy the three conditions in the previous paragraph; for example, divisibility is transitive — that is, if a is a factor of b and b is a factor of c, then a is a factor of c. This of course has to be proved.

Example (1) is different from the other three examples in that given any two real numbers a and b either a is related to b or b is related to a. That's not true in example (2) since, for example, neither 3 is a factor of 11 nor is 11 a factor of 3. Nor is it true in example (3) since for example neither of {a,b} and {b,c} is a subset of the other. Nor is it true in example (4), since you can eat breakfast and get dressed in either order.

A partial ordering R of A where given any two elements a,b of A, either <a,b> or <b,a> is in R is called a "total ordering". In a total ordering every two elements can be compared. A total ordering is also called a "linear ordering" since the elements of A can, so to speak, be arranged in a line.

The standard orderings of the natural numbers, of the integers, of the rational numbers, and the real numbers are all examples of linear orderings, as is the natural ordering of the numbers from 1 to n (for any n). These orderings are all different from one another.

There is essentially only one way to order a finite set, but there are many ways of ordering an infinite set. For example, the natural numbers can be ordered by placing all the odd numbers before all the even numbers, so that the ordering looks like 1, 3, 5, 7, 9, 11, … 2, 4, 6, 8, 10, 12, … .

This ordering is difficult to picture since most people approach this example with two biases: one is that you can't order the numbers this way because 2 belongs between 1 and 3; the other is a personal notion of infinity that excludes having 2 come after an infinite sequence of elements.

Mathematicians are able to overcome those biases, to break those rules and visualize things in different ways than civilians do.

Thus there are many different ways of placing a linear order on the natural numbers (or other sets); the number of different linear orderings on the natural numbers is actually uncountable. It is the study of such linear orderings that is the subject of "Linear Orderings".

Is a monomial a polynomial?

Yes, a monomial is a polynomial.

In ordinary English, "mono" and "poly" have opposite meanings, as in monogamy vs. polygamy, monotheism vs. polytheism, monolingual vs. polylingual, etc.

However, mathematicians have their own language and, at least in this case, every monomial is also considered a polynomial.

In this situation it makes things much simpler to consider monomials as polynomials. For example, if you wanted to make a statement about all polynomials, you would have to instead make that statement about all monomials and all polynomials, so you would have to give two proofs, one that worked for all monomials and another that worked for all polynomials that weren't monomials. That's extra work, and mathematicians would not be fond of it.

A similar example: Are squares also rectangles? You could argue it either way. But if all squares are considered rectangles, then if you're proving a theorem about all rectangles, you would avoid having two separate cases — one for squares and another for rectangles that are not squares. So in the mathematical world, all squares are rectangles (at least in English).

Another example of how words have a different definition in mathematics than in ordinary speech: When you are asked for a definition of a term, you look in the dictionary and there are multiple ways of defining that term. In mathematics, however, each term must have a single definition. So the word "definition" is different in mathematics than it is in ordinary speech.

What proven math fact surprised you the most when you learned it?

One mathematical fact that continues to surprise me is the "Fundamental Theorem of Arithmetic", even though I have explained the proof many times to classes of college juniors who intend to be math majors. Though it is called "fundamental", most people have never heard of it, although every high school student really should know about it (even if they are not prepared to understand its proof).

There are two parts to the theorem. The first is that every positive integer can be written as a product of prime numbers. That's something that many students learn about in school, sort of; for example, they can write 100 as 2*5*2*5.

You might object and say that 11 is not a product of primes. But, from the point of view of mathematicians, 11 is the "product" of one prime, namely 11; so we, by convention, agree that every prime is considered a "product" of primes.

The second part of the theorem is that every number can be written as a product of primes in a unique way. That's not quite true, because 6 can be written both as 2*3 and 3*2. However, if we ignore the order in which the primes are multiplied, every positive integer can indeed be written in a unique way as a product of primes.

That to me remains surprising. I would like to think that there is some renegade number, maybe with 50 digits, that can be factored in two completely different ways. Why is that so unreasonable?

It is not at all obvious that there is no such number, and it would be wonderful if some number refused to conform to the Fundamental Theorem of Arithmetic, but surprisingly that can't happen.

To the best of scientists and mathematicians guess, how many areas of math are yet to be discovered?

The answer is limited only by human imagination. By the way, your use of the word "discovered" assumes that, like relics of ancient civilizations, mathematicians will discover new areas of math by turning up the rocks that cover them. Another view, perhaps held by most mathematicians [can we take a vote?] is that a mathematician "creates" new mathematics, like a composer creates a symphony, not like an archaeologist who "discovers" a new shard.

What made the Pythagoreans number abstract concepts?

When teaching one of my first classes many years ago, I asked my class how many of them had been at a zoo and had seen a "three"? Of course none of them had, although many of them had seen a zebra.

The point is that numbers are essentially abstract. We see three apples and three children and recognize that the two collections share the property of "three-ness."

Is three-ness in our minds, is it a social convention, or is it part of God's grand design, or all three — however you look at it, it's an abstraction.

Four-ness, on the other hand, …