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The Role of Teachers in Mathematics Education Reform

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ABSTRACT. In discussions of the state of mathematics education today, teachers are often viewed as being part of the problem. More attention should be focused on how teachers are and can be an important part of the solution. Drawing on programs directed by the presenter, this paper will discuss some of the roles for teachers in mathematics education reform.

The above abstract, prepared several months before this paper, appears to fit both as an introduction to and as a summary of this paper.

I

First of all, when we mathematicians think about mathematics education, our view of high school mathematics is typically that the high school teacher is part of the problem.

What else should we think when the students we see as first-year undergraduates are lacking in the mathematical skills we think they should have. Why can't our students add fractions or remember the rules for exponentiation? Why can't they identify, let alone sketch, the graphs of basic functions? Why do they have such trouble with word problems, and why don't they recognize that their answers to word problems are often simply nonsense?

"Obviously," many of our colleagues conclude, "it must be the fault of their high school teachers. They had these students in their classes just last year. Why didn't they teach them these topics?"

The strange thing is that when senior-level high school teachers get together they ask the same questions—but they address them to teachers of intermediate algebra... and we can guess what happens when intermediate algebra teachers get together!

Everyone agrees that many students do not achieve mastery in mathematics, but the causes are complex; we could easily enumerate a half-dozen

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differences between the 1990s and previous decades which would help to explain the phenomenon. Focusing simply on the high school teachers as the cause of the problem is an example of the *post hoc ergo propter hoc* fallacy we have all learned to avoid.

There are, of course, many high school teachers of mathematics whose training is not adequate, often through no fault of their own. They may have been pressed into service due to a shortage of mathematics teachers in their states—or due to a surplus of social studies teachers in their school systems. They may be graduates of teacher training programs which emphasized pedagogy and all but ignored mathematics. They may have returned to teaching—or to mathematics—after many years elsewhere. Or they may simply have not learned mathematics well.

But there are also many expert high school mathematics teachers. While I cannot make any statistical claims about their numbers, as a result of my activities during the past five years, I know that there are many. I have worked with them on a number of projects and programs. Their enthusiasm for mathematics matches ours, and their desire and ability to communicate that enthusiasm usually surpasses ours.

Unfortunately, these teachers are not in the public eye; the reports on education focus on the shortcomings of mathematics education, not on its strengths.

Expert teachers are an important, often untapped resource in the reform of mathematics education. In this article, I will describe several models of how their expertise can be used, as well as how the academic and research mathematics communities can facilitate that process.

II

Not only is society unaware of the existence of expert mathematics teachers, we mathematicians do not recognize their existence either.

The split in the mathematical community between high school and college-level teachers has caused a great deal of harm to both. There may never have been a time when college teachers really saw high school teachers as colleagues,but for the last few academic generations, there has been no awareness among academic mathematicians that the two groups share a common background and a common agenda.

How often does it happen that high school and college mathematics teachers sit down together and discuss, as colleagues, their common concerns about mathematics education?

For most of us, the only encounters we have with high school mathematics teachers is at parents' night at the high school, and our typical reaction is that we would do a better job of teaching our kids.

We are often disappointed when we find that high school teachers are not as knowledgeable or skilled in mathematics as we are. We forget that we need

them to be teachers, not research mathematicians, and that their training is not the same as ours. We also assume, incorrectly, that because we know mathematics better, we can teach it better.

Consequently, the proposals that come from the college community are often prescriptive. "They don't really know about mathematics and how to teach it. We'll show them how to do it right!" is a common attitude among our colleagues. Another is "Let's get better mathematics students to become high school teachers!" Such proposals don't go very far in building a collegial atmosphere. That's not to say that we shouldn't upgrade the skills of teachers and that we shouldn't encourage more good mathematics students to become teachers, but if that's all we say, then we project a perspective that is bound to be counter-productive.

We have to start, instead, with the assumption that many high school teachers of mathematics share our enthusiasm for mathematics and share our agenda for reform and improvement in mathematics education.

We have to start with the assumption that high school teachers often know as much, or even more, about the problems than do practicing mathematicians.

We also have to accept the reality that changing the teaching of mathematics in the schools can only be effected by teachers in the schools.

We must recognize that high school teachers must have important roles in mathematics education reform and use our resources and our stature to help them bring about that reform.

We must also encourage and support their efforts and work toward real partnerships in mathematics eduction reform, where the pronouns "we" and "they" are no longer used in a self-congratulating or pejorative way.

III

Three examples of such partnerships will highlight three important roles that teachers can have in mathematics education reform—in training inexperienced teachers, in disseminating new approaches, and in cooperative efforts in improving instruction.

These roles are highlighted, respectively, in the Institute for New Mathematics Teachers, the Leadership Program in Discrete Mathematics, and the Precalculus Project, three programs I direct at Rutgers University. Brief descriptions of these programs are provided here; further information can of course be obtained from me directly.

These programs are sponsored by the Rutgers University Center for Mathematics, Science, and Computer Education. The Center was founded in 1985 to work with schools in the areas of mathematics, science, and computer education. Directed by Gerald A. Goldin, the Center sponsors research, curriculum development, summer institutes, partnerships with school districts, conferences, lecture series, and in-service educational programs. The Center

has been funded since its inception by Rutgers University and the New Jersey State Department of Higher Education.

One very successful program involves using expert mathematics teachers to help train new mathematics teachers. The Institute for New Mathematics Teachers (and a parallel institute for new science teachers) is a week-long residential institute which takes place each August, with follow-up sessions during the school year. The program has been funded for the last three years by the New Jersey Department of Higher Education using Title-II funds, now called Eisenhower funds.

Participants bring with them their teaching assignments for September and review with the staff, in small homogeneous groups, the textbooks and curriculum guidelines they will be using for their classes. The staff provides the new teachers with suggestions for how they can supplement the material and for how they can motivate their students to learn. The title of the institute is Teaching Excellent Mathematics, and that is a goal that we hope will ultimately be achieved by the ninety new mathematics teachers who have participated in the first three years of the program.

The staff consists of high school and middle school teachers and supervisors of science and mathematics who are selected because of their own reputations for excellence in teaching.

Here are a few reasons why excellent practicing teachers are particularly appropriate for this type of role:

- They have collected and developed materials over the years which they have used in their courses, and they are eager to share these materials with the new teachers.
- They have been able to show their own students that "mathematics is not a spectator sport," and they have been experimenting with current ideas and techniques such as "discovery learning," "problem-solving," "hands-on manipulatives," and "cooperative learning" to enhance student involvement in the learning process.
- They are enthusiastic about mathematics and caring about their students and want to communicate their enthusiasm to their students.
- They serve as excellent role models in instruction; they are able to involve the participants in the program in the same ways that they would like the participants in the program to involve their own students.
- They are mentors who can explain their own commitments as teachers of mathematics and can provide the new teachers with assistance and support in becoming expert teachers.

We have been truly fortunate that each staff member selected for our program has indeed been an excellent selection, and when you think in terms of the varied roles that the staff play in the program, you can see how important this has been for the success of the program.

I'm certain that there are many other teachers who would also have been excellent selections. I have learned that **there are many teachers who are really *under-utilized*, that the structures of schools and school systems do not provide these expert teachers with opportunities to share their expertise.**

We need to identify these teachers and help create such opportunities for them. We mathematicians and our institutions are in a position to make such opportunities happen; because of our reputations, we can attract the participation of expert teachers as staff, of learning teachers as participants, and of state and federal agencies as financial supporters.

Let me expand on this point. Suppose that you decide to organize a series of workshops for high school teachers, and that you set about to identify and select the best teachers in your area as staff for the program. These teachers would want to participate because their selection by a college or university to play such roles is itself welcome and unexpected recognition of their efforts and their expertise. Other teachers would want to participate because your institution has a reputation in your state, and both the participants and their supervisors would recognize the potential value of the program. Your institution would value the program as an opportunity for both public service and public relations, and the funding agencies will be interested because you are providing an opportunity that no one else can.

A brief discussion of another example: As part of our Precalculus Project, about which I will say more shortly, we offer a summer institute for precalculus teachers. We find that high school teachers often spend many years teaching lower-level courses before they have an opportunity to teach precalculus—often when a precalculus teacher retires. By that time, they need a real conceptual review of the material and an opportunity to explore how the apparently disparate pieces of precalculus fit together. The summer institute provides such an overview of precalculus to twenty-five teachers each year, and, to a large extent, it is staffed by expert high school teachers.

In these examples, teachers teach other teachers about traditional components of the high school curriculum, but **the principle of teachers teaching teachers is applicable also when new areas and focuses of mathematics are discussed.**

Let us assume that the community has decided that a new topic—perhaps probability and statistics, or discrete mathematics, or mathematical modeling, or linear algebra—or a new technology should receive increased emphasis in the schools. How should we carry out that decision? Simple arithmetic shows that even if each of us were to conduct a program for teachers, the number of teachers we could reach would be very few indeed.

In order to involve many schools in a new direction, teachers have to be enlisted to instruct additional teachers. That means that we should not focus exclusively on upgrading the skills of individual teachers—as did National Science Foundation (NSF) funded projects in the 1960s—but to work with those teachers who can most effectively carry the message to other teachers.

This is one focus of another program that we offer at Rutgers University. Early in 1989, the NSF awarded a grant to create a Science and Technology Center in Discrete Mathematics and Theoretical Computer Science (DIMACS); located at Rutgers University, DIMACS is a consortium of Rutgers and Princeton Universities, AT&T Bell Laboratories, and Bell Communications Research. As part of its educational program, DIMACS provides support for an institute in discrete mathematics for high school teachers. The first program, entitled Networks and Algorithms, and funded by DIMACS, took place in the summer of 1989; an expanded version of the pilot program, called the Leadership Program in Discrete Mathematics, was offered in the summer of 1990 with financial support from the NSF.

The purpose of the Leadership Program in Discrete Mathematics is to create a leadership cadre of high school teachers. We expect program participants to become knowledgeable about concepts of discrete mathematics and their applications. We also expect them to develop materials and activities for incorporating these concepts into their classes and to introduce their colleagues to these materials and activities.

We cannot, of course, assume that participants will come to the institute knowledgeable about discrete mathematics, but we can ensure, through the recruitment and selection process, that the participants are teachers who will be able to play leadership roles in disseminating information to colleagues both in their schools and in other schools.

During the institute, participants study the subject matter and applications of discrete mathematics. They then work together to create materials and lessons which they will use in their own classes. At follow-up sessions they discuss their activities with each other and report to the group on their progress. Once they have successfully incorporated institute topics in their own classes, they find suitable forums, within their districts and beyond, for disseminating the topics to other teachers.

Although the pilot program has not yet been completed, it is clear that it has been successful. Essentially all of the participants have indeed incorporated institute materials into their classes; many have been preparing curriculum materials based on what they have done; and many have given, or are planning to give workshops for other teachers.

throughout the region on how to introduce discrete mathematics into the high school curriculum. This model and the theme of "teachers teaching teachers" was developed for the Woodrow Wilson Institutes that take place each summer at Princeton University under the sponsorship of the Woodrow Wilson National Fellowship Foundation.

The theme of the Institutes for New Mathematics Teachers is to use experienced teachers to teach new teachers, and the theme of the Leadership Program in Discrete Mathematics is to use lead teachers to disseminate new approaches. A third program, the Precalculus Project, has a different theme altogether—that of teachers working together to improve instruction. The Precalculus Project grew out of one of the first activities sponsored by the Rutgers University Center for Mathematics, Science, and Computer Education.

Consistent with its mission, the Center's first event was a conference in March 1985 attended by mathematics, science, and computer educators from around New Jersey.

Those attending the conference were asked in which areas they particularly sought our assistance and involvement, and the mathematics teachers and supervisors responded that one area which needed particular attention was precalculus. Indeed, when fifteen high school teachers and mathematics chairs gathered subsequently, they discovered that each of their schools had a precalculus course, but that no two of their courses were the same. The group formed the Precalculus Committee and set about determining what students needed in order to be properly prepared for calculus.

After a year of monthly meetings, the Precalculus Committee produced a report called *The Core Precalculus Course*, which described in some detail what students needed to understand and be able to do in order to succeed in the first two semesters of calculus. Recognizing that different schools and different types of students have different needs, the committee did not attempt to prescribe a standardized precalculus course; rather, the committee tried to describe what the "core" of every precalculus course should be. In some situations, that core could be completed in one semester and supplemented by additional material; whereas, in others, a full year might not even suffice. The report was disseminated throughout New Jersey and has been used by many districts in reviewing and revising their mathematics curricula.

Since the focus of the article is not on precalculus but on the role of high school teachers, I will return to the main topic. As the committee proceeded with its deliberations, I found that the teachers were attending our late afternoon sessions regularly, sometimes braving the New Jersey elements to do so. I asked myself why were they so devoted; why were they coming to

collegiality. If we have a mathematical concern, we simply walk down the hall and bounce our ideas off our colleagues; or we have our discussion over lunch; or we call a meeting.

A high school teacher often does not have colleagues. He or she may be the only precalculus teacher in the school, or, equivalently, the only precalculus teacher who cares about mathematics or about education. Even if there are other teachers, their schedules may preclude any conversation other than brief exchanges between classes. Lunch is a time for cafeteria duty, not for professional talk, and departmental meetings are usually devoted to administrative, rather than educational concerns.

As a result, many high school teachers do not have opportunities to engage in the kind of conversations that the Precalculus Committee offered—conversations that deal with mathematics and how one can provide a setting where students will be motivated to think mathematically.

I found that our discussions of the core precalculus course were providing exactly this opportunity, so we have followed this up with other opportunities. For example, we have a test item bank committee, which has been working for several years on creating a collection of precalculus questions based on the core precalculus course recommendations. The committee distributed the first version of the "test item bank" two years ago and each year since has worked on improving and extending various sections of the original test item bank. The meetings of the committee, while focused on developing a "product," provide the participants with interesting discussions on precalculus topics. At a recent meeting, the following questions were discussed: What kinds of inequalities should we expect students to be able to solve? What techniques do we expect them to use? If we emphasize the interrelationship between the geometric and algebraic perspectives, should we spend more time on solving inequalities graphically? Where else can teachers discuss such questions!

Each year we sponsor a day-long conference entitled Good Ideas in Teaching Precalculus which is attended by over 200 New Jersey high school and college teachers of precalculus. At each of five sessions, participants can choose to attend one of about eight programs, many of them presentations by high school teachers. At each session, we also have "idea exchanges"—opportunities for teachers to engage each other in discussion on a particular topic—for example, introducing limits in precalculus. In both the presentations and the idea exchanges, teachers have an opportunity to discuss mathematical concerns with their colleagues.

This spring we are enlarging our program to create several new workshop

will formulate an objective to be achieved in these four meetings. Two types of benefits will be achieved through these meetings. The participants will have an opportunity to discuss mathematics and the teaching of mathematics with their colleagues, and the community will benefit because what each group produces will be disseminated to other teachers in the state.

This year we are also planning to introduce several more intensive workshops, involving five full days of activity. For example, at any given time there are many districts that are revising their precalculus curricula, or that are considering districtwide implementation of the NCTM *Standards*, or that are considering the introduction of new material in the precalculus course. Why not bring people from these districts together so that they can benefit from joint discussions about curriculum revision (or one of the other topics)? Why not have this group draw up a handbook of issues that need to be addressed in revising a precalculus curriculum?

Each of these activities can be considered as an opportunity provided, under university auspices, for high school personnel to have professional discussions which will be useful for themselves and for the broader community.

The Precalculus Project has grown from a small committee to a mailing list of over 1000 New Jersey precalculus teachers who have participated in our programs or requested our materials. It is now an independently funded program, supported by Title II (Eisenhower) funds provided by the New Jersey State Department of Higher Education. Many high school teachers are actively interested in improving mathematics education and in working and networking with their colleagues. The Precalculus Project provides them many opportunities for doing so.

IV

Drawing on programs I direct at Rutgers University, I have described three roles for high school teachers in mathematics education reform—in training inexperienced teachers, in disseminating new approaches, and in cooperative efforts in improving instruction.

I hope that I have argued convincingly that there are dedicated and enthusiastic teachers who are interested in playing such roles, and that providing opportunities for teachers to do so is a worthwhile endeavor.

It is an endeavor to which I have devoted most of my time and efforts in the last few years. After writing a number of articles in model theory and recursive function theory and a book on *Linear Orderings*, I served as Director of the Undergraduate Program in Mathematics at Rutgers Univer-

appropriate assumption seemed to me that high school teachers would be allies, not adversaries, in the process. As I became involved in various activities, I repeatedly found evidence that my assumption was indeed appropriate. So it is not surprising that I am now organizing opportunities for teachers to bring about change in their instruction and in their schools.

Organizing such opportunities is not a simple task and may be much more than most readers of this article are willing to undertake. Nevertheless, there are an increasing number of mathematicians who are involved in such organizational efforts and who would welcome the interest and participation of their colleagues. In any case, there are two tasks with which I would like to charge all of my readers.

The first is to counter the negative remarks that are continually directed at high school teachers; don't let your colleagues get away with gratuitous potshots at the schools! Don't let them generalize from personal anecdotes to a general indictment of all high schools and all teachers. Such generalizations are academically dishonest and clearly counterproductive; like other prejudices, they should be opposed vigorously. Positive change can only come about if we and our colleagues recognize that high school and college teachers of mathematics have similar problems and similar goals.

The second is to find ways to provide support and encouragement to teachers. Visit your local schools and sit in on some classes; you may even be invited to try your hand at conducting a high school class. Be supportive—even though you may not feel supportive. Acknowledge that the teacher's job is not an easy one and offer your encouragement. Recognize that the curriculum can be improved and encourage the school to explore alternatives, but be aware that you may not have the background to be sure that your solutions are the right ones. Be particularly cautious about recommending broad changes if your main goal is to improve your own child's education; what you think is best for your child may not in fact advance the common good (and may not even be best for your child). Show that you care and appreciate the teachers' efforts, that you are willing to serve as a resource and a colleague, and that you are willing to put your time and thought into a common search for solutions.

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